REAL REGULAR REFINING OPTIONS:
A Study for MBS Real Options[1] NOV 2012

Questions

1. What is the mean and volatility of the 5:3:2 crack spread over 2009-2011? What is the appropriate payoff for NER as function of crude oil and refined product prices, if it is possible to shut-down and restart the refinery?
2. What are the mean, volatility and correlation parameter values for the 2H 2009, 2010, 2011, and 1H 2012 for NER ?
3. What are the optimal shut-down and restart spread thresholds, and how are these affected by changes in the expected volatility of the inputs, outputs or correlation?
4. Peter might observe four end period sets of crude oil and refined product prices, and NER stockmarket prices, as in Table 3. When is the refinery overvalued/undervalued in the stockmarket according to PV or ROV? Suggest appropriate actions for Peter to issue equity or repurchase shares each end period, and evaluate the risks of such recommended actions.

 The model is based on parts of "Real input-output energy-switching options" *Journal of Energy Markets*, Vol. 5 Fall 2012, pages 1-20, by Roger Adkins and Dean Paxson. Some of the production and crack data is not entirely dissimilar to that of Alon Refining Krotz Springs Inc as disclosed in the 10-K for the year ended December 2011.

1 Introduction

Peter Botts is the newly appointed Chief Real Options Manager (CROM) of National Energy Refinery (NER), a limited partnership newly quoted on the NYSE. He is aware that part of the success of NER is going to be his ability to manage the shut-down and re-start of operations of the refinery as the crack spread changes. The flexibility of NER means that, when handled appropriately, the refinery could be considered a call option on the spread, operating at full capacity when the spread widens. Peter's previous experience had been with a European refinery for the last five months in 2009 with a benchmark 5:3:2 crack spread, based on Brent crude and US refining products. NER's benchmark is 5:3:2, based on Louisiana Light Sulfur (LLS) crude oil, Unleaded 87 gasoline and Ultra Low Sulfur Diesel (ULSD). LLS often trades at a premium to the benchmark US WTI, similar to Brent crude oil, which is used as a proxy.

When is the right time for an operator of a flexible facility to switch back and forth between two possible commodity outputs in order to maximise value when operating and switching costs are taken into account? Which factors should be monitored in making these decisions? How much should an investor pay for such a flexible operating asset? What are the strategy implications for the operator, investor and possibly policy makers?

2 Multiple Input-Output Switching

2.1 Assumptions

Consider a flexible facility which can be used to produce an output (refined products) using one input (crude oil), but by incurring a switching cost can be shut down, and by incurring another switching cost can be restarted. Assume the prices of the output, x and the input y, are stochastic and correlated and follow geometric Brownian motion:

$$dx = (\mu_x - \delta_x) x\, dt + \sigma_x\, x\, dz_x \qquad (1)$$

$$dy = (\mu_y - \delta_y) y\, dt + \sigma_y\, y\, dz_y \qquad (2)$$

with the notations:

μ	Required return on the output/input
δ	Convenience yield of the output/input
σ	Volatility of the output/input
dz	Wiener processes
ρ	Correlation between the input and output prices: $dz_x\, dz_y / dt$
c	Other operating costs

The instantaneous cash flow in the operating mode is the respective commodity price of the output less unit input plus any other operating cost, assuming production of one (equivalent) unit per annum. Other operating costs "c" are per unit produced, for convenience are assumed proportional to y, so y is multiplied by (1+c). A switching cost of S_{12} is incurred when switching from operating mode '1' to the non-operating mode '2', and S_{21} for switching back. The riskless interest rate is r.

Further assumptions are that the lifetime of the facility is infinite, and the company is not restricted in the shut down/start up choice because of selling or buying commitments. Moreover, the typical assumptions of real options theory apply, with interest rates, convenience yields, volatilities and correlation constant over time.

2.2 Quasi-analytical Solution for Multiple Input-Output Switching

The asset value with opportunities to continuously switch between an operating mode and a suspended mode (when both inputs and outputs are stochastic) is given by the present value of perpetual cash flows in the current operating mode plus the option to switch to the alternative mode. Let V_1 be the asset value in operating mode '1', producing output x at input cost y, and V_2 the asset value in a suspension mode '2'. The switching options depend on the two correlated stochastic variables x and y, and so do the asset value functions, which are defined by the following partial differential equations(PDE):

(3)

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_1}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_1}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_1}{\partial x \partial y} + (r-\delta_X)x\frac{\partial V_1}{\partial x} + (r-\delta_Y)y\frac{\partial V_1}{\partial y} - rV_1 + (x - y*(1+c)) = 0$$

(4)

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_2}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_2}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_2}{\partial x \partial y} + (r-\delta_X)x\frac{\partial V_2}{\partial x} + (r-\delta_Y)y\frac{\partial V_2}{\partial y} - rV_2 = 0$$

The operating mode has the production income (x-y) (y having been multiplied by 1+c) and the option to suspend; the suspension mode has only the option to re-start operations. For stochastic outputs and inputs, the PDEs are satisfied by the following general solutions:

$$V_1(x,y) = \frac{x}{\delta_x} - \frac{y}{\delta_y} + A x^{\beta_{11}} y^{\beta_{12}} \quad (5)$$

where β_{11} and β_{12} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{11}(\beta_{11}-1) + \tfrac{1}{2}\sigma_y^2\beta_{12}(\beta_{12}-1) + \rho\sigma_x\sigma_y\beta_{11}\beta_{12} + \beta_{11}(r-\delta_x) + \beta_{12}(r-\delta_y) - r = 0, \quad (6)$$

and

$$V_2(x,y) = B x^{\beta_{21}} y^{\beta_{22}} \quad (7)$$

where β_{21} and β_{22} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{21}(\beta_{21}-1) + \tfrac{1}{2}\sigma_y^2\beta_{22}(\beta_{22}-1) + \rho\sigma_x\sigma_y\beta_{21}\beta_{22} + \beta_{21}(r-\delta_x) + \beta_{22}(r-\delta_y) - r = 0$$

(8)

The characteristic root equation (6) is solved by combinations of β_{11} and β_{12} forming an ellipse of such form that β_{11} could be positive or negative and β_{12} could be positive or negative. The same is true for equation (8). Since the option to switch from operating to suspension decreases with x and increases with y, β_{11} must be negative and β_{12} positive. Likewise, β_{21} must be positive and β_{22} negative. Switching between the operating and suspension modes always depends on the level of both x and y. At the switching points (x_{12}, y_{12}) (shut down) and (x_{21}, y_{21}) (start up), the asset value in the current mode must be equal to the asset value in the alternative mode net of switching costs. These value matching conditions are stated formally below:

$$V_1(x_{12}, y_{12}) = V_2(x_{12}, y_{12}) - S_{12}$$

$$V_2\left(x_{21}, y_{21}\right) = V_1(x_{21}, y_{21}) - S_{21}$$

$$A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}} + \frac{x_{12}}{\delta_x} - \frac{y_{12}}{\delta_y} - \frac{c}{r} = B x_{12}^{\beta_{21}} y_{12}^{\beta_{22}} - S_{12} \tag{9}$$

$$B x_{21}^{\beta_{21}} y_{21}^{\beta_{22}} = A x_{21}^{\beta_{11}} y_{21}^{\beta_{12}} + \frac{x_{21}}{\delta_x} - \frac{y_{21}}{\delta_y} - S_{21} \tag{10}$$

Furthermore, smooth pasting conditions hold at the boundaries:

$$\beta_{11} A x_{12}^{\beta_{11}-1} y_{12}^{\beta_{12}} + \frac{1}{\delta_x} = \beta_{21} B x_{12}^{\beta_{21}-1} y_{12}^{\beta_{22}} \tag{11}$$

$$\beta_{12} A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}-1} - \frac{1}{\delta_y} = \beta_{22} B x_{12}^{\beta_{21}} y_{12}^{\beta_{22}-1} \tag{12}$$

$$\beta_{21} B x_{21}^{\beta_{21}-1} y_{21} = \beta_{11} A x_{21}^{\beta_{11}-1} y_{21}^{\beta_{12}} + \frac{1}{\delta_x} \tag{13}$$

$$\beta_{22} B x_{21}^{\beta_{21}} y_{21}^{\beta_{22}-1} = \beta_{12} A x_{21}^{\beta_{11}} y_{21}^{\beta_{12}-1} - \frac{1}{\delta_y} \tag{14}$$

There are only 8 equations, (6) and (8-14), for 10 unknowns, β_{11}, β_{12}, β_{21}, β_{22}, A, B, x_{12}, y_{12}, x_{21}, y_{21}, so there is no completely analytical solution. Yet, for every value of x, there has to be a corresponding value of y when switching should occur, (x_{12}, y_{12}) and (x_{21}, y_{21}). So a quasi-analytical solution can be found by assuming values for x, which then solves the set of simultaneous equations for all remaining variables, given that $x = x_{12} = x_{21}$. This procedure is repeated for many values of x, providing the corresponding option values and the switching boundaries. This set of simultaneous equations can be solved using Excel.

3. Empirical Application: Valuing a Refinery

The continuous input-output option model is applied to determine the real value of a flexible refinery, with shut-down and restart costs.

3.1. Econometric model for the input & output stochastic processes

The real regular refining option model is based on the assumption that commodity prices follow geometric Brownian motion. An analysis of the time series over the last half of 2009 reveals an annual volatility of 32% for the input and 34% for the portfolio of two outputs. It can also be seen from Figure 1 that the price movements are slightly more marked for gasoline. Furthermore, the figure suggests a high correlation between the inputs and outputs. The correlation between Brent crude oil and the weighted portfolio of refined products is 0.89.

Figure 1. Prices of NER Inputs and Outputs

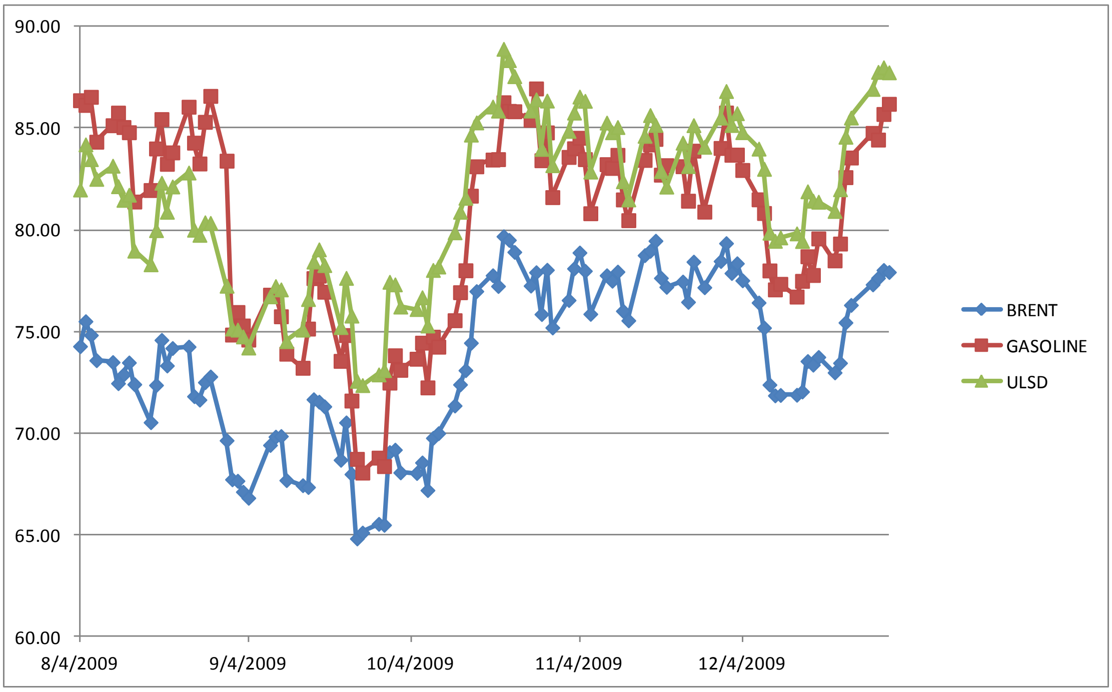


Table 1. Econometric Analysis of Commodity Prices

	BRENT	GASOLINE	ULSD	PRODUCTS	
VOL	**0.322**	0.382	0.321	**0.344**	(DIRECT)
CORREL	**0.891**	0.840		0.344	PORTFOLIO

Product (.6 Gasoline + .4 ULSD) volatility is .344, correlation of Gasoline and ULSD is .840, so Portfolio volatility is SQRT((.6^2)*(.382^2)+(.4^2)*(.321^2)+2*.84*.6*.4*.382*.321)=.344.

Figure 2. Time Series of 5:3:2 CRACK SPREAD

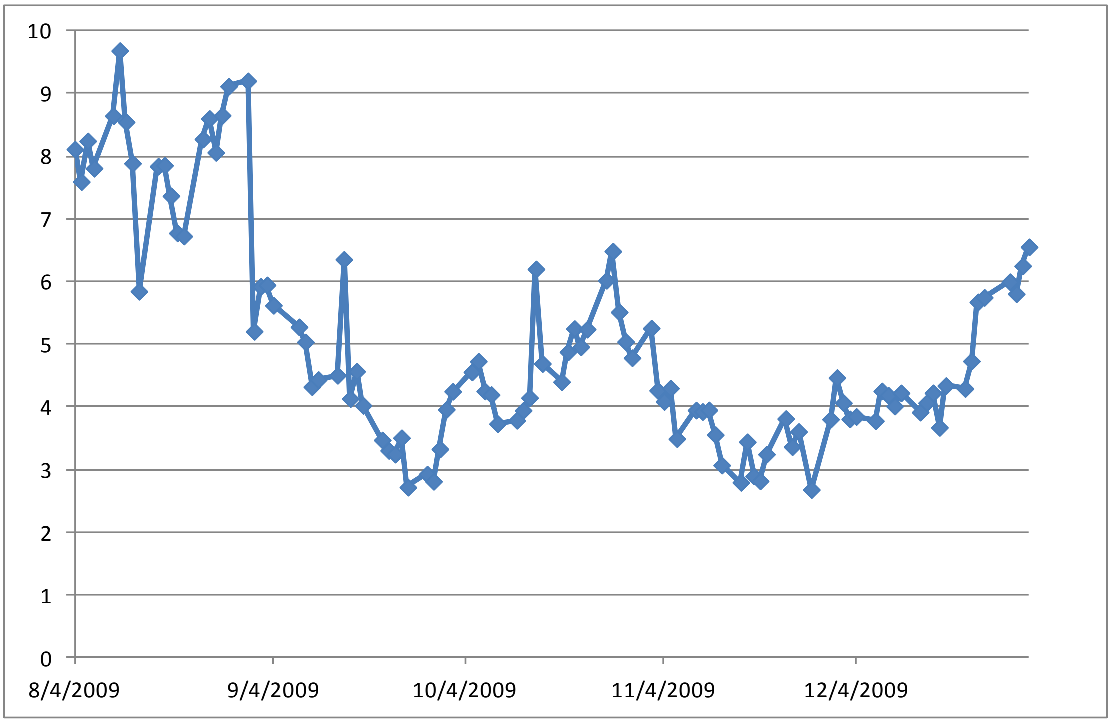


Figure 2 shows a 5:3:2 Full Crack Spread (Brent Crude Oil, NYMEX Gasoline and ULSD) from August 2009 through December 2009, with an operating expense of 3% of the input cost. During mid-November the spread became very low, but near the end of the period, as US refined products began to reflect the higher spread between Brent and WTI, the crack spread increased.

3.2. Asset-specific parameters

The key characteristics of the refinery are given in Table 2 and 3 together with the calculation of the operating margin based on the parameter values as of December 2009.

Table 2. Parameter Values & Switching Thresholds for the Real Regular Refinery

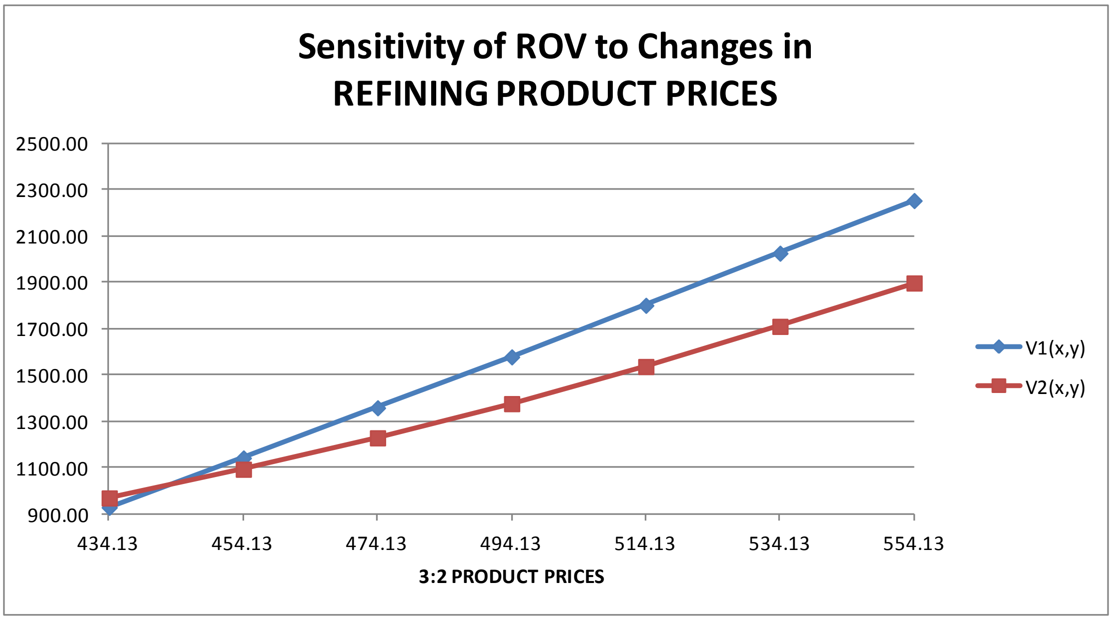
	A	B	C	D	E	F	G	H	I
1	**MULTIPLE INPUT-OUTPUT SWITCHING OPTION**				END DEC 2009				
2		INPUT	**c=**	0.03	**BRENT**	**GASOLINE**	**ULSD**		
3	PRICE	x	434.13	3*F3+2*G3	77.93	86.21	87.75		
4	OPERATING COST	y (1+c)	401.34	5*E3*(1+J3)		**CRACK**	**FULLCRACK**		
5	Convenience yield of x	δ_x	0.08			44.48	32.79		
6	Convenience yield of y	δ_y	0.08						
7	Volatility of x	σ_x	0.34						
8	Volatility of y	σ_y	0.32						
9	Correlation x with y	ρ	0.89						
10	Risk-free interest rate	r	0.09						
11	Switching cost from x to y	S_{12}	1,340						
12	Switching cost from y to x	S_{21}	400						
13	Switching boundary OP to SHUT	$\mathbf{x_{12}}$	**434**						
14	Switching boundary SHUT to OP	$\mathbf{x_{21}}$	**434**						
15		SOLUTION		OPTION	OPERATION				
16	Asset value in operating mode	$V_1(x,y)$	926.16	516.27	409.88				
17	Asset value in shut down mode	$V_2(x,y)$	974.55	974.55					
18		A	0.10						
19		B	0.41						
20	Switching boundary OP to SHU	$\mathbf{y_{12}(x)}$	**774.40**						
21	Switching boundary SHUT to O	$\mathbf{y_{21}(x)}$	**269.02**						
22	Solution quadrant	β_{11}	-1.36457						
23	Solution quadrant	β_{12}	2.80318						
24	Solution quadrant	β_{21}	2.83662						
25	Solution quadrant	β_{22}	-1.57802						
26		EQUATIONS							
27	Value matching 1	EQ 9	0.000	(C18*C13^C22*C20^C23+C13/C5-C20/C6-C19*C13^C24*C20^C25+C11)					
28	Value matching 2	EQ 10	0.000	(C19*C14^C24*C21^C25-C18*C14^C22*C21^C23-C14/C5+C21/C6+C12)					
29	Smooth pasting 1A	EQ 11	0.000	(C22*C18*(C13^(C22-1))*C20^C23+1/C5-C24*C19*(C13^(C24-1))*C20^C25)					
30	Smooth pasting 1B	EQ 12	0.000	(C23*C18*C13^C22*(C20^(C23-1))-1/C6-C25*C19*C13^C24*(C20^(C25-1)))					
31	Smooth pasting 2A	EQ 13	0.000	(C24*C19*(C14^(C24-1))*C21^C25-C22*C18*(C14^(C22-1))*C21^C23-1/C5)					
32	Smooth pasting 2B	EQ 14	0.000	(C25*C19*C14^C24*(C21^(C25-1))-C23*C18*C14^C22*(C21^(C23-1))+1/C6)					
33	Solution quadrant 1	EQ 6	0.000	0.5*C7^2*C22*(C22-1)+0.5*C8^2*C23*(C23-1)+C9*C7*C8*C22*C23+C22*(C10-C5)+C23*(C10-C6)-C10					
34	Solution quadrant 2	EQ 8	0.000	0.5*C7^2*C24*(C24-1)+0.5*C8^2*C25*(C25-1)+C9*C7*C8*C24*C25+C24*(C10-C5)+C25*(C10-C6)-C10					
35		Sum	0.000	**Solver**					
36	SWITCH SPREAD		505.38						
37	Asset value in operating mode	EQ 5	(C18*(C3^C22)*(C4^C23)+C3/C5-C4/C6)						
38	Asset value in shut down mode	EQ 7	(C19*C3^C24*C4^C25)						

The variable cost of production is composed of natural gas and other materials used in the refining process, logistics cost for the delivery of the final product, personnel cost and maintenance costs.

3.3. Asset valuation

Considering first the operating state, when the plant is operated and the spread is earned, it can be seen that the asset value (AV_1) increases linearly in p for very high levels of p. Figure 3 shows the sensitivity of the ROV operating and suspended to changes in the CRACK SPREAD, as product prices are increased while input prices remain constant. The lower and upper switching boundaries increase with the crack spread, as the difference between the boundaries also increases.

Figure 3



3.4. Firm valuation

Peter believed he deserves an incentive for excellence in performance as a CROM, but also imagined that sometimes his company might be over or under-valued in the stockmarket. If overvalued, perhaps a share issue should be considered; if undervalued, shares might be repurchased. Other assets of the company consisting mostly of receivables from selling refined products, some crude and refined product inventories, and cash were likely to remain at around $50,000,000, there are no liabilities, and 100,000,000 outstanding shares, as in Table 3 below. Peter wondered if the real option value exceeds the present value, especially if the crack spread is low, stockmarket investors may be basing decisions on valuation methods developed in the late 19th and early 20th centuries like present values. If so, perhaps shares might be repurchased when the real option value significantly exceeds the stockmarket value, and re-issued when the opposite holds.

Table 3

	A	B	C	D	E
1	**STOCKMARKET VS REAL OPTION VALUE**				
2		Dec-09	Dec-10	Dec-11	Jun-12
3	MARKET PRICE	20	55	15	55
4	SHARES	100,000,000			
5	MARKET CAPITALISATION	$2,000,000,000	$5,500,000,000	$1,500,000,000	$5,500,000,000
6	MC -OTHER ASSETS	$1,950,000,000	$5,495,000,000	$1,495,000,000	$5,495,000,000
7		DAILY			
8	BRENT INPUT	77.93			
9	GASOLINE	86.21			
10	ULSD	87.75			
11	CRACK per BBL	8.89			
12	FULL CRACK per BBL	6.56			
13	CAPACITY	80,000			
14	CAPACITY USE	0.7			
15	PER UNIT TOTAL VALUE	185.23			
16	OPERATING VALUE	81.98			
17		ANNUALIZED			
18	REFINERY THROUGHPUT	20,440,000			
19	GROSS REFINERY INCOME	$1,774,701,365			
20	NET REFINERY INCOME	$134,047,564			
21		VALUE: V-OA			
22	NET REF INC/δ	$1,675,594,550			
23	REAL OPTION VALUE	$3,786,125,144			
24	PRESENT VALUE	$1,675,594,550			
25					
26	PER UNIT TOTAL VALUE	OPTION!$C$16/5			
27	OPERATING VALUE	OPTION!$E$16/5			
28	REFINERY THROUGHPUT	365*B13*B14			
29	GROSS REFINERY INCOME	B18*(3*B9+2*B10)/5			
30	NET REFINERY INCOME	B12*B18			
31	NET REF INC/δ	B20/OPTION!$C$5			
32	REAL OPTION VALUE	B18*B15			
33	PRESENT VALUE	B18*B16			

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